**7x+11y = 100**

**Investigation**

Write 100 as the sum of two integers, one divisible by 7 and the other divisible by 11. Use your answer to find formulas giving all the solutions of the following equation where x and y are integers.

**So: 7x + 11y = 100**

First of all I made a table to try and find the different solutions to the problem. In the 1st column I listed the multiples of 7. Then in the 3rd column I put the number that when u add it to the number in the 1st column makes 100. If the number in the 3rd column is a multiple of 11 then that is one of the solutions. In the 2nd and 4th column I marked the places where I found solutions to the equation. I split up the table to show different situations where the integers for x and y are either positive or negative.

First I found integers for **7x + 11y = 100** where both x and y are positive.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **X** |  | **y** |
| 98 |  | 2 |  |
| 91 |  | 9 |  |
| 84 |  | 16 |  |
| 77 |  | 23 |  |
| 70 |  | 30 |  |
| 63 |  | 37 |  |
| 56 | 8 | 44 | 4 |
| 49 |  | 51 |  |
| 42 |  | 58 |  |
| 35 |  | 65 |  |
| 28 |  | 72 |  |
| 21 |  | 79 |  |
| 14 |  | 86 |  |
| 7 |  | 93 |  |
| 0 |  | 100 |  |

Then I found integers where x is positive and y is negative

|  |  |  |  |
| --- | --- | --- | --- |
|  | **X** |  | **y** |
| 364 | 52 | -264 | -24 |
| 357 |  | -257 |  |
| 350 |  | -250 |  |
| 343 |  | -243 |  |
| 336 |  | -236 |  |
| 329 |  | -229 |  |
| 322 |  | -222 |  |
| 315 |  | -215 |  |
| 308 |  | -208 |  |
| 301 |  | -201 |  |
| 294 |  | -194 |  |
| 287 | 41 | -187 | -17 |
| 280 |  | -180 |  |
| 273 |  | -173 |  |
| 266 |  | -166 |  |
| 259 |  | -159 |  |
| 252 |  | -152 |  |
| 245 |  | -145 |  |
| 238 |  | -138 |  |
| 231 |  | -131 |  |
| 224 |  | -124 |  |
| 217 |  | -117 |  |
| 210 | 30 | -110 | -10 |
| 203 |  | -103 |  |
| 196 |  | -96 |  |
| 189 |  | -89 |  |
| 182 |  | -82 |  |
| 175 |  | -75 |  |
| 168 |  | -68 |  |
| 161 |  | -61 |  |
| 154 |  | -54 |  |
| 147 |  | -47 |  |
| 140 |  | -40 |  |
| 133 | 19 | -33 | -3 |
| 126 |  | -26 |  |
| 119 |  | -19 |  |
| 112 |  | -12 |  |
| 105 |  | -5 |  |

Then I found integers where x is negative and y is positive:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **X** |  | **y** |
| -7 |  | 107 |  |
| -14 |  | 114 |  |
| -21 | -3 | 121 | 11 |
| -28 |  | 128 |  |
| -35 |  | 135 |  |
| -42 |  | 142 |  |
| -49 |  | 149 |  |
| -56 |  | 156 |  |
| -63 |  | 163 |  |
| -70 |  | 170 |  |
| -77 |  | 177 |  |
| -84 |  | 184 |  |
| -91 |  | 191 |  |
| -98 | -14 | 198 | 18 |
| -105 |  | 205 |  |
| -112 |  | 212 |  |
| -119 |  | 219 |  |
| -126 |  | 226 |  |
| -133 |  | 233 |  |
| -140 |  | 240 |  |
| -147 |  | 247 |  |
| -154 |  | 254 |  |
| -161 |  | 261 |  |
| -168 |  | 268 |  |
| -175 | -25 | 275 | 25 |
| -182 |  | 282 |  |
| -189 |  | 289 |  |
| -196 |  | 296 |  |
| -203 |  | 303 |  |
| -210 |  | 310 |  |
| -217 |  | 317 |  |
| -224 |  | 324 |  |
| -231 |  | 331 |  |
| -238 |  | 338 |  |
| -245 |  | 345 |  |
| -252 | -36 | 352 | 32 |
| -259 |  | 359 |  |
| -266 |  | 366 |  |
| -273 |  | 373 |  |
| -280 |  | 380 |  |
| -287 |  | 387 |  |

I then put my results in order and looked for a pattern

**When x and y are positive:** in this one, since there was only one set of positive integers that worked, I could not find a rule.

|  |  |  |
| --- | --- | --- |
| (positive)x |  | (positive)y |
| 8 |  | 4 |

**When x is positive and y is negative:** here, since I have more solutions, I can find a rule

|  |  |  |
| --- | --- | --- |
| (positive) x | | (negative)y |
| 19 |  | -3 |
| 30 |  | -10 |
| 41 |  | -17 |
| 52 |  | -24 |

x:

19 , 30 , 41 , 52

+ 11 +11 +11

11 22 33 44

19-11 = 8

30-22 = 8

41-33 = 8

52 – 44 = 8

The rule is:

11n+8

First I found the rule for when x is positive, the difference between each integer for x was 11. So the start of the rule had to be 11n. Then I multiplied 11 by the nth term for each number, e.g. the nth term for 19 is 1, the nth term for 30 is 2, etc. Finally I found the difference between the integer for x and the integer’s nth term multiplied by 11 and the difference for all of them was 8. So the rule for when x is positive is 11n+8.

y:

-3 , -10 , -17 , -24

-7 -7 -7

-7 -14 -21 -28

-3 - -7 = 4

-10 - -14 = 4

-17 - -21= 4

-24 - -28 = 4

The rule is

-7n+4

Then I found the rule for when y is negative, the difference between each integer for y is -7. So the start of the rule is -7n. Then I multiplied -7 by the nth term for each number e.g. the nth term for -3 is 1, the nth term for -10 is 2, etc. Finally I found the difference between the integer for y and the integer’s nth term multiplied by -7 and the difference for all of them was 4. So the rule for when y is negative is -7n+4

So if we put the two rules into the equation **7x+ 11y = 100** then we would get:

7(11n+8) + 11(-7n+4) = 100

This rule will work with any number used in the nth term to give you a solution where x is positive and y is negative.

To test to make sure the equation is correct I checked it using the 100th term:

the 100th term:

|  |
| --- |
| 7[11(100)+8] +11 [-7(100)+4] = 100 |
| 7(1100+8) + 11(-700+4)=100 |
| 7(1108) + 11(-696) = 100 |
| 7756+-7656= 100 |
| 7756-7656= 100 |

This test shows that the equation is correct.

**When x is negative and y is positive:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (negative) x | |  | (positive)y |  |
| -3 |  |  |  | 11 |
| -14 |  |  |  | 18 |
| -25 |  |  |  | 25 |
| -36 |  |  |  | 32 |

-3 , -14 , -25 , -36

-11 -11 -11

-11 -22 -33 -44

x:

-3 - -11 = 8

-14 - -22 = 8

-25 - -33 = 8

-36 - -44 = 8

The rule is:

-11n + 8

This time I found the rule for when x is negative, the difference between each integer for x was -11. So the start of the rule had to be -11n. Then I multiplied -11 by the nth term for each number, e.g. the nth term for -3 is 1, the nth term for -14 is 2, etc. Finally I found the difference between the integer for x and the integer’s nth term multiplied by -11 and the difference for all of them was 8. So the rule for when x is negative is -11n+8.

11 , 18 , 25 , 32

+7 +7 +7

7 14 21 28

y:

11 – 7 = 4

18 – 14 = 4

25 – 21 = 4

32 – 28 = 4

The rule is:

7n + 4

Then I found the rule for when y is positive, the difference between each integer for y is 7. So the start of the rule is 7n. Then I multiplied 7 by the nth term for each number e.g. the nth term for 11 is 1, the nth term for 18 is 2, etc. Finally I found the difference between the integer for y and the integer’s nth term multiplied by 7 and the difference for all of them was 4. So the rule for when y is positive is 7n+4

So if we put the two rules into the equation **7x+ 11y = 100** then we would get:

7(-11n+8) + 11(7n+4) = 100

This rule will work with any number used in the nth term to give you a solution where x is negative and y is positive.

To test to make sure the equation is correct I checked it using the 100th term:

|  |
| --- |
| 100th term: |
| 7[-11 (100)+8] + 11[7(100)+4] = 100 |
| 7(-1100+8) + 11(700+4) = 100 |
| 7(-1092)+11(704) = 100 |
| -7644+7744= 100 |
|  |

This test shows that the equation is correct.

**Conclusion:**

In this investigation I found two rules and one anomaly that didn’t fit either rule (7(8)+ 11(4) = 100) because both integers were positive. Looking at the table used to find the integers that fit the equation where the anomaly is, along with the other two sets: above and below; we can see that the two rules start from the anomaly. When we increase the value of x, the value of y decreases and viseversa. So the anomaly fits into both rules if the nth term for the anomaly is 0:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **x** |  | **y** |
| 133 | 19 | -33 | -3 |
| 126 |  | -26 |  |
| 119 |  | -19 |  |
| 112 |  | -12 |  |
| 105 |  | -5 |  |
| 98 |  | 2 |  |
| 91 |  | 9 |  |
| 84 |  | 16 |  |
| 77 |  | 23 |  |
| 70 |  | 30 |  |
| 63 |  | 37 |  |
| 56 | 8 | 44 | 4 |
| 49 |  | 51 |  |
| 42 |  | 58 |  |
| 35 |  | 65 |  |
| 28 |  | 72 |  |
| 21 |  | 79 |  |
| 14 |  | 86 |  |
| 7 |  | 93 |  |
| 0 |  | 100 |  |
| -7 |  | 107 |  |
| -14 |  | 114 |  |
| -21 | -3 | 121 | 11 |